Решение транспортной задачи:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  | | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | Запасы | | *B*1 | *B*2 | *B*3 | | *A*1 | |  | | --- | | 3 | | |  | | --- | | 2 | | |  | | --- | | 1 | | |  | | --- | | 100 | | | *A*2 | |  | | --- | | 1 | | |  | | --- | | 1 | | |  | | --- | | 4 | | |  | | --- | | 250 | | | *A*3 | |  | | --- | | 2 | | |  | | --- | | 3 | | |  | | --- | | 3 | | |  | | --- | | 200 | | | Потребности | |  | | --- | | 100 | | |  | | --- | | 250 | | |  | | --- | | 200 | | |  | | --- | | 0 | | |

Число пунктов отправления *m=*3, а число пунктов назначения *n*=3. Следовательно опорный план задачи определяется числами, стоящими в *m+n*−1=3+3−1=5 заполненых клетках таблицы. Тарифы перевозок единицы груза из каждого пункта отправления во все пункты назначения задаются матрицей

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | *C=* |  | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  | | --- | | 3 | | |  | | --- | | 2 | | |  | | --- | | 1 | | | |  | | --- | | 1 | | |  | | --- | | 1 | | |  | | --- | | 4 | | | |  | | --- | | 2 | | |  | | --- | | 3 | | |  | | --- | | 3 | | |  | |

Наличие груза у поставщиков равно:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ∑ *A*i= | |  | | --- | | 100 | | + | |  | | --- | | 250 | | + | |  | | --- | | 200 | | = | |  | | --- | | 550 | |  |

Общая потребность в грузе в пунктах назначения равна:

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| ∑ *B*i= | |  | | --- | | 100 | | + | |  | | --- | | 250 | | + | |  | | --- | | 200 | | = | |  | | --- | | 550 | |  |

∑ *A*i=∑ *B*i. Модель транспортной задачи является закрытой. Следовательно она разрешима.

**Этап I. Нахождение первого опорного плана**

Найдем опорный план задачи *методом северно-западного* угла.

*A*1≤*B*1. Следовательно в клетку (*A*1, *B*1 ) помещаем число *min*(*A*1, *B*1 )=100. Запасы пункта *A*1 полностью исчерпаны. Поэтому исключаем из рассмотрения строку *A*1 и будем считать потребности пункта *B*1 равными 100−100=0.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | | *A*1 | |  | | --- | | 3 | |  | |  | | --- | | 2 | |  | |  | | --- | | 1 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 100 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 100 | |  |  | | | *A*2 | |  | | --- | | 1 | |  | |  | | --- | | 1 | |  | |  | | --- | | 4 | |  | |  | | --- | | 250 | | |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 250 | |  |  | | | *A*3 | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 3 | |  | |  | | --- | | 200 | | |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 200 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 250 | | | |  | | --- | | 200 | | | |  | | --- | | 550 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 100 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 250 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 200 | |  |  | | | |

*A*2>*B*1. Следовательно в клетку (*A*2, *B*1) помещаем число *min*(*A*2, *B*1)=0. Потребности пункта *B*1 полностью удовлетворены. Поэтому исключаем из рассмотрения столбец *B*1 и будем считать запасы пункта *A*2 равными 250−0=250.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | | *A*1 | |  | | --- | | 3 | |  | |  | | --- | | 2 | |  | |  | | --- | | 1 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 100 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 100 | |  |  | | | *A*2 | |  | | --- | | 1 | |  | |  | | --- | | 1 | |  | |  | | --- | | 4 | |  | |  | | --- | | 250 | | |  | |  | | --- | | 0 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 250 | |  |  | | | *A*3 | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 3 | |  | |  | | --- | | 200 | | |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 200 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 250 | | | |  | | --- | | 200 | | | |  | | --- | | 550 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 100 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 250 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 200 | |  |  | | | |

*A*2≤*B*2. Следовательно в клетку (*A*2, *B*2 ) помещаем число *min*(*A*2, *B*2 )=250. Запасы пункта *A*2 полностью исчерпаны. Поэтому исключаем из рассмотрения строку *A*2 и будем считать потребности пункта *B*2 равными 250−250=0.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | | *A*1 | |  | | --- | | 3 | |  | |  | | --- | | 2 | |  | |  | | --- | | 1 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 100 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 100 | |  |  | | | *A*2 | |  | | --- | | 1 | |  | |  | | --- | | 1 | |  | |  | | --- | | 4 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 0 | |  | |  | | --- | | 250 | |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 250 | |  |  | | | *A*3 | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 3 | |  | |  | | --- | | 200 | | |  |  |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 200 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 200 | | | |  | | --- | | 550 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 100 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 250 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 200 | |  |  | | | |

*A*3>*B*2. Следовательно в клетку (*A*3, *B*2) помещаем число *min*(*A*3, *B*2)=0. Потребности пункта *B*2 полностью удовлетворены. Поэтому исключаем из рассмотрения столбец *B*2 и будем считать запасы пункта *A*3 равными 200−0=200.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | | *A*1 | |  | | --- | | 3 | |  | |  | | --- | | 2 | |  | |  | | --- | | 1 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 100 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 100 | |  |  | | | *A*2 | |  | | --- | | 1 | |  | |  | | --- | | 1 | |  | |  | | --- | | 4 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 0 | |  | |  | | --- | | 250 | |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 250 | |  |  | | | *A*3 | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 3 | |  | |  | | --- | | 200 | | |  |  |  | |  | | --- | | 0 | |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 200 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 200 | | | |  | | --- | | 550 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 100 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 250 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 200 | |  |  | | | |

*A*3≤*B*3. Следовательно в клетку (*A*3, *B*3 ) помещаем число *min*(*A*3, *B*3 )=200. Запасы пункта *A*3 полностью исчерпаны. Поэтому исключаем из рассмотрения строку *A*3 и будем считать потребности пункта *B*3 равными 200−200=0.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | | *A*1 | |  | | --- | | 3 | |  | |  | | --- | | 2 | |  | |  | | --- | | 1 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 100 | |  |  |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 100 | |  |  | | | *A*2 | |  | | --- | | 1 | |  | |  | | --- | | 1 | |  | |  | | --- | | 4 | |  | |  | | --- | | 0 | | |  | |  | | --- | | 0 | |  | |  | | --- | | 250 | |  |  | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 250 | |  |  | | | *A*3 | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 3 | |  | |  | | --- | | 0 | | |  |  |  | |  | | --- | | 0 | |  | |  | | --- | | 200 | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 200 | |  |  | | | Потребности | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 0 | | | |  | | --- | | 550 | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 100 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 250 | |  |  | | | |  |  |  |  |  | | --- | --- | --- | --- | --- | |  | |  | | --- | | 200 | |  |  | | | |

**Этап II. Улучшение опорного плана**

Найдем оптимальный план транспортной задачи *методом потенциалов*.

Опорный план имеет следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *X=* |  | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  | | --- | | 100 | | |  | | --- | | 0 | | |  | | --- | | 0 | | | |  | | --- | | 0 | | |  | | --- | | 250 | | |  | | --- | | 0 | | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 200 | | |  |  |

При этом плане стоимость перевозок вычисляется так:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | S= | 3 | · | 100 | + | 1 | · | 0 | + | 1 | · | 250 | + | 3 | · | 0 | + | 3 | · | 200 | = | 1150 | |  |

Проверяем полученный опорный план на оптимальность. Для этого находим потенциалы пунктов отправления и назначения. Для заполненных клеток составляем систему из 5 уравнений с 6 неизвестными:

* β1−α1=3
* β1−α2=1
* β2−α2=1
* β2−α3=3
* β3−α3=3

Полагая α1=0, находим β1=3 α2=2 β2=3 α3=0 β3=3 .

Для каждой свободной клетки вычисляем число αij=βj−αi−cij:

α12=1, α13=2, α23=-3, α31=1.

Полученные числа заключаем в рамки и записываем их в соотвестствующие клетки таблицы:

.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | | *A*1 | |  | | --- | | 3 | |  | |  | | --- | | 2 | |  | |  | | --- | | 1 | |  | |  | | --- | | 100 | | |  | |  | | --- | | 100 | |  | |  |  | | --- | --- | | |  | | --- | | 1 | | |  | |  |  | | --- | --- | | |  | | --- | | 2 | | | | *A*2 | |  | | --- | | 1 | |  | |  | | --- | | 1 | |  | |  | | --- | | 4 | |  | |  | | --- | | 250 | | |  | |  | | --- | | 0 | |  | |  | | --- | | 250 | |  | |  |  | | --- | --- | | |  | | --- | | −3 | | | | *A*3 | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 3 | |  | |  | | --- | | 200 | | |  | |  |  | | --- | --- | | |  | | --- | | 1 | | |  | |  | | --- | | 0 | |  | |  | | --- | | 200 | | | Потребности | |  | | --- | | 100 | | | |  | | --- | | 250 | | | |  | | --- | | 200 | | | |  | | --- | | 550 | | |

Среди чисел *α*ij есть положительные. Следовательно данный опорный план не является оптимальным. Наибольшее положительное число 2 находится в пересечении строки *A*1 и столбца *B*3. Для данной свободной клетки строим цикл пересчета. Для этого вставим в эту клетку знак "+" а остальные клетки цикла поочередно знаки "−" и "+".

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | | *A*1 | |  | | --- | | 3 | | − | |  | | --- | | 2 | |  | |  | | --- | | 1 | | + | |  | | --- | | 100 | | |  | |  | | --- | | 100 | |  | |  |  | | --- | --- | | |  | | --- | | 1 | | |  | |  |  | | --- | --- | | |  | | --- | | 2 | | | | *A*2 | |  | | --- | | 1 | | + | |  | | --- | | 1 | | − | |  | | --- | | 4 | |  | |  | | --- | | 250 | | |  | |  | | --- | | 0 | |  | |  | | --- | | 250 | |  | |  |  | | --- | --- | | |  | | --- | | −3 | | | | *A*3 | |  | | --- | | 2 | |  | |  | | --- | | 3 | | + | |  | | --- | | 3 | | − | |  | | --- | | 200 | | |  | |  |  | | --- | --- | | |  | | --- | | 1 | | |  | |  | | --- | | 0 | |  | |  | | --- | | 200 | | | Потребности | |  | | --- | | 100 | | | |  | | --- | | 250 | | | |  | | --- | | 200 | | | |  | | --- | | 550 | | |

Наименьшее из чисел в минусовых клетках равно 100. Клетка, в которой находится это число становится свободной. В новой таблице другие числа получаются так. Числам, находящимся в плюсовых клетках добавляется 100, а из чисел, находящихся в минусовых клентках вычитается это число.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | | *A*1 | |  | | --- | | 3 | |  | |  | | --- | | 2 | |  | |  | | --- | | 1 | |  | |  | | --- | | 100 | | |  |  |  |  |  | |  | | --- | | 100 | | | *A*2 | |  | | --- | | 1 | |  | |  | | --- | | 1 | |  | |  | | --- | | 4 | |  | |  | | --- | | 250 | | |  | |  | | --- | | 100 | |  | |  | | --- | | 150 | |  |  | | *A*3 | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 3 | |  | |  | | --- | | 200 | | |  |  |  | |  | | --- | | 100 | |  | |  | | --- | | 100 | | | Потребности | |  | | --- | | 100 | | | |  | | --- | | 250 | | | |  | | --- | | 200 | | | |  | | --- | | 550 | | |

Опорный план имеет следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *X=* |  | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 100 | | | |  | | --- | | 100 | | |  | | --- | | 150 | | |  | | --- | | 0 | | | |  | | --- | | 0 | | |  | | --- | | 100 | | |  | | --- | | 100 | | |  |  |

При этом плане стоимость перевозок вычисляется так:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | S= | 1 | · | 100 | + | 1 | · | 100 | + | 1 | · | 150 | + | 3 | · | 100 | + | 3 | · | 100 | = | 950 | |  |

Проверяем полученный опорный план на оптимальность. Для этого находим потенциалы пунктов отправления и назначения. Для заполненных клеток составляем систему из 5 уравнений с 6 неизвестными:

* β3−α1=1
* β1−α2=1
* β2−α2=1
* β2−α3=3
* β3−α3=3

Полагая α1=0, находим β3=1 α3=-2 β2=1 α2=0 β1=1 .

Для каждой свободной клетки вычисляем число αij=βj−αi−cij:

α11=-2, α12=-1, α23=-3, α31=1.

Полученные числа заключаем в рамки и записываем их в соотвестствующие клетки таблицы:

.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | | *A*1 | |  | | --- | | 3 | |  | |  | | --- | | 2 | |  | |  | | --- | | 1 | |  | |  | | --- | | 100 | | |  | |  |  | | --- | --- | | |  | | --- | | −2 | | |  | |  |  | | --- | --- | | |  | | --- | | −1 | | |  | |  | | --- | | 100 | | | *A*2 | |  | | --- | | 1 | |  | |  | | --- | | 1 | |  | |  | | --- | | 4 | |  | |  | | --- | | 250 | | |  | |  | | --- | | 100 | |  | |  | | --- | | 150 | |  | |  |  | | --- | --- | | |  | | --- | | −3 | | | | *A*3 | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 3 | |  | |  | | --- | | 200 | | |  | |  |  | | --- | --- | | |  | | --- | | 1 | | |  | |  | | --- | | 100 | |  | |  | | --- | | 100 | | | Потребности | |  | | --- | | 100 | | | |  | | --- | | 250 | | | |  | | --- | | 200 | | | |  | | --- | | 550 | | |

Среди чисел *α*ij есть положительные. Следовательно данный опорный план не является оптимальным. Наибольшее положительное число 1 находится в пересечении строки *A*3 и столбца *B*1. Для данной свободной клетки строим цикл пересчета. Для этого вставим в эту клетку знак "+" а остальные клетки цикла поочередно знаки "−" и "+".

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | | *A*1 | |  | | --- | | 3 | |  | |  | | --- | | 2 | |  | |  | | --- | | 1 | |  | |  | | --- | | 100 | | |  | |  |  | | --- | --- | | |  | | --- | | −2 | | |  | |  |  | | --- | --- | | |  | | --- | | −1 | | |  | |  | | --- | | 100 | | | *A*2 | |  | | --- | | 1 | | − | |  | | --- | | 1 | | + | |  | | --- | | 4 | |  | |  | | --- | | 250 | | |  | |  | | --- | | 100 | |  | |  | | --- | | 150 | |  | |  |  | | --- | --- | | |  | | --- | | −3 | | | | *A*3 | |  | | --- | | 2 | | + | |  | | --- | | 3 | | − | |  | | --- | | 3 | |  | |  | | --- | | 200 | | |  | |  |  | | --- | --- | | |  | | --- | | 1 | | |  | |  | | --- | | 100 | |  | |  | | --- | | 100 | | | Потребности | |  | | --- | | 100 | | | |  | | --- | | 250 | | | |  | | --- | | 200 | | | |  | | --- | | 550 | | |

Наименьшее из чисел в минусовых клетках равно 100. Клетка, в которой находится это число становится свободной. В новой таблице другие числа получаются так. Числам, находящимся в плюсовых клетках добавляется 100, а из чисел, находящихся в минусовых клентках вычитается это число.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | | *A*1 | |  | | --- | | 3 | |  | |  | | --- | | 2 | |  | |  | | --- | | 1 | |  | |  | | --- | | 100 | | |  |  |  |  |  | |  | | --- | | 100 | | | *A*2 | |  | | --- | | 1 | |  | |  | | --- | | 1 | |  | |  | | --- | | 4 | |  | |  | | --- | | 250 | | |  | |  | | --- | | 0 | |  | |  | | --- | | 250 | |  |  | | *A*3 | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 3 | |  | |  | | --- | | 200 | | |  | |  | | --- | | 100 | |  |  |  | |  | | --- | | 100 | | | Потребности | |  | | --- | | 100 | | | |  | | --- | | 250 | | | |  | | --- | | 200 | | | |  | | --- | | 550 | | |

Опорный план имеет следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *X=* |  | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 100 | | | |  | | --- | | 0 | | |  | | --- | | 250 | | |  | | --- | | 0 | | | |  | | --- | | 100 | | |  | | --- | | 0 | | |  | | --- | | 100 | | |  |  |

При этом плане стоимость перевозок вычисляется так:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | S= | 1 | · | 100 | + | 1 | · | 0 | + | 1 | · | 250 | + | 2 | · | 100 | + | 3 | · | 100 | = | 850 | |  |

Проверяем полученный опорный план на оптимальность. Для этого находим потенциалы пунктов отправления и назначения. Для заполненных клеток составляем систему из 5 уравнений с 6 неизвестными:

* β3−α1=1
* β1−α2=1
* β2−α2=1
* β1−α3=2
* β3−α3=3

Полагая α1=0, находим β3=1 α3=-2 β1=0 α2=-1 β2=0 .

Для каждой свободной клетки вычисляем число αij=βj−αi−cij:

α11=-3, α12=-2, α23=-2, α32=-1.

Полученные числа заключаем в рамки и записываем их в соотвестствующие клетки таблицы:

.

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | | Пункты  отправления | Пункты назначения | | | | | | Запасы | | *B*1 | | *B*2 | | *B*3 | | | *A*1 | |  | | --- | | 3 | |  | |  | | --- | | 2 | |  | |  | | --- | | 1 | |  | |  | | --- | | 100 | | |  | |  |  | | --- | --- | | |  | | --- | | −3 | | |  | |  |  | | --- | --- | | |  | | --- | | −2 | | |  | |  | | --- | | 100 | | | *A*2 | |  | | --- | | 1 | |  | |  | | --- | | 1 | |  | |  | | --- | | 4 | |  | |  | | --- | | 250 | | |  | |  | | --- | | 0 | |  | |  | | --- | | 250 | |  | |  |  | | --- | --- | | |  | | --- | | −2 | | | | *A*3 | |  | | --- | | 2 | |  | |  | | --- | | 3 | |  | |  | | --- | | 3 | |  | |  | | --- | | 200 | | |  | |  | | --- | | 100 | |  | |  |  | | --- | --- | | |  | | --- | | −1 | | |  | |  | | --- | | 100 | | | Потребности | |  | | --- | | 100 | | | |  | | --- | | 250 | | | |  | | --- | | 200 | | | |  | | --- | | 550 | | |

Среди чисел *α*ij нет положительных. Следовательно данный опорный план является оптимальным.

**Решение:**

Оптимальный план имеет следующий вид:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| *X=* |  | |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | | |  | | --- | | 0 | | |  | | --- | | 0 | | |  | | --- | | 100 | | | |  | | --- | | 0 | | |  | | --- | | 250 | | |  | | --- | | 0 | | | |  | | --- | | 100 | | |  | | --- | | 0 | | |  | | --- | | 100 | | |  |  |

При этом плане стоимость перевозок вычисляется так:

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  | | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | | S= | 1 | · | 100 | + | 1 | · | 0 | + | 1 | · | 250 | + | 2 | · | 100 | + | 3 | · | 100 | = | 850 | |

Ответ: При оптимальном плане стоимость будет: 850